The Relationship between Students' Attitudes Towards Mathematics and their Mathematical Achievement According to TIMSS 2015: A Comparative Study Among Saudi Arabia, South Korea, Singapore, and the United States of America

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Abstract: This study utilized the mathematics data from the Trends in International Mathematics and Science Study (TIMSS 2015) to investigate the effects of students' attitude scales (students' views on engaging in mathematics lessons, students like learning mathematics, students' confident in mathematics, students' value mathematics) on 8th-grade student mathematics achievement in Korea, Saudi Arabia, Singapore, and the United States. According to the F-test findings showed that for each scale of attitudes towards mathematics, students who reported being engaged in, liking, feeling confident in, and valuing mathematics had higher average mathematics scores than their counterparts in four countries. However, the multiple regression model showed a different direction of result which indicated that students' views on engagement in mathematics lessons had relatively little negative effect on their mathematics achievement in TIMSS 2015. However, a positive effect on achievement was seen in Singaporean 8th-grade students who reported that they like learning mathematics and in Korean 8th-grade students who reported that they value mathematics. In four countries, 8th-graders' confidence in mathematics variable was found to be a significant predictor for their mathematics achievement.

Keywords: student attitudes towards mathematics; mathematics achievement; TIMSS 2015.

اتجاهات طلاب الصف الثامن نحو الرياضيات وتأثيرها على تحصيلهم الرياضي وفقًا لنتائج TIMSS 2015 : دراسة مقارنة بين كوريا الجنوبية والمملكة العربية السعودية وسنغافورا والولايات المتحدة الأمريكية

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الملخص: استخدمت هذه الدراسة نتائج الطلاب في دراسات التوجهات الدولية في العلوم والرياضيات (TIMSS 2015) للكشف عن تأثير اتجاهات الطلاب (الاندماج في دروس الرياضيات، حب تعلم الرياضيات، الثقة، تقدير الرياضيات) في تحصيلهم الرياضي لصف الثامن في كل من كوريا الجنوبية والمملكة العربية السعودية وسنغافورة والولايات المتحدة الامريكية. حيث أظهرت نتائج اختبار F ان الطلاب الذين اندمجوا في تعليم الرياضيات وأعجبوا بها وشعروا بالثقة نحوها واعطوها اهمية حصلوا على درجات في اختبار الرياضيات أعلى من نظرائهم في جميع الدول المشاركة. وأظهرت نتائج الانحدار المتعدد اتجاهًا مختلفًا للنتيجة والتي اشارت إلى أن متغير اندماج الطلاب في دروس الرياضيات كان له تأثير سلبيا نسبيًا على تحصيلهم الرياضي. الا انه كان هنالك تأثير إيجابيا على تحصيل طلاب الصف الثامن في سنغافورا بالنسبة لطلاب الذين أفادوا بأنهم يحبون الرياضيات وتأثيرا إيجابيا على تحصيل طلاب الصف الثامن في كوريا الذين اشاروا أنهم يقدرون أهمية الرياضيات. كما كشفت الدراسة على ان متغير ثقة الطلاب بأنفسهم في فهم الرياضيات كان مؤشرًا إيجابيا على تحصيل الطلاب في مادة الرياضيات في جميع دول المقارنة.

الكلمات المفتاحية: اتجاهات الطلاب نحو الرياضيات، التحصيل الرياضي، TIMSS 2015.

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Introduction

International large-scale assessments are essential criteria for evaluating the quality of education in many countries, given the breadth and depth of the information they provide. Multiple methods for evaluating promising educational policies and practices can help specialists make informed and correct decisions in this regard.

During the past two decades, many countries have participated in international studies of educational evaluation and quality control. One of the most important data sources for these is the Trends in International Mathematics and Science Study (TIMSS), an assessment of mathematics and science achievement of fourth- and eighth-grade students. TIMSS is organised by the International Association for the Evaluation of Educational Achievement (IEA) and has been administered every four years since 1995. TIMSS 2015 was the sixth in the series, providing comprehensive data on student performance, mathematics and science learning contexts, and educational attainment trends (Mullis, Drucker, Preuschoff, Arora, & Stanco, 2016).

TIMSS provides comparative information about achievement and other factors, such as the characteristics of students, their attitudes towards learning, school environments, and teachers, for the purpose of improving teaching and learning mathematics and science. In addition, it provides each country with information related to student achievement during successive TIMSS rounds. Finally, it also provides each country with rich and varied resources about factors affecting achievement that may contribute to the enhancement of the educational process (Knight, P., Lietz, P., Best, Nugroho, D. M., & Tobin, M, 2012).

Many studies have been conducted to investigate the effects of several variables on student achievement on international assessments such as TIMSS and PISA, including those related to the student, the teacher, and the school environment (Ababneh, 2019; Al-Ghamdi, 2010; Al-Hajjaji, 2012; Guven, 2017; Phan, 2008; Yi, H., & Lee, Y, 2017; Shehadeh & Al-Qarmati, 2016). Affective variables are important in mathematics, especially at such a critical point as eighth grade. Therefore, such variables should be addressed, most notably students' confident in mathematics, like learning mathematics, engagement in mathematics lessons, and value mathematics (Yavuz, Demirtasli, Yalcin, & Dibek, 2017). The relationship between students' attitudes towards mathematics as a whole and their achievement in this subject should also be considered (Spencer, 2012).

The participation of the Kingdom of Saudi Arabia, in addition to the other countries of the world in TIMSS represents a milestone in the educational system, given the rich and varied data provided by TIMSS on the quality of the educational systems. In addition, the results of these tests provide opportunities for comparison

between different countries. In terms of achievement, by examining the impact of group of factors on achievement and comparing their impact among the participating countries (Mullis, Martin, Foy, & Hooper, 2016). In this regard, three countries were selected (South Korea, Singapore and the United States) along with the Kingdom of Saudi Arabia. They were chosen for several reasons, most notably the diversity in achievement levels. Where the performance level of eighth grade students in the Kingdom of Saudi Arabia was classified within the lowest level, as the average performance of eighth grade students in mathematics was 368 points (Al-Shamrani et al., 2016), while Singapore and South Korea were classified within the high level with average scores of 621 and 606 in mathematics, respectively. Another reason of choosing these two countries is because that they are two Asian countries which English is not their mother tongue, as is the case of Saudi Arabia which makes it legitimate to compare these two countries with Saudi Arabia as a comparison knowing that the content domains of these countries aligned with the content domains of TIMSS mathematics assessment. The United States is also ranked at an intermediate level, ranking tenth with an average score of 518 in mathematics. In addiction to be a reference country in the series of mathematics curricula applied in the Kingdom of Saudi Arabia (McGraw-Hill Curriculum Series).

Significance of the Study

The significance of the study stems from its reliance on the outcomes of students' mathematics achievement provided by TIMSS 2015, to reveal the most influential features in mathematics achievement for eighth graders by comparing the results of the participating countries.

Theoretical Background

The theoretical background of this study is supported by a model called the "success cycle" by Koshy, Ernest & Casey (2009), which investigated the relationship between positive impacts of attitudes and mathematical achievement and found that students with positive attitudes toward math had the tendency to press more effort and persistence which encouraged students' willingness to complete math tasks. These variables interact with each other's and form a cycle. Students with higher mathematical self-confidence, sense of mathematical self-efficacy, positive attitudes and motivation toward mathematics tend to do more effort and persistence of doing mathematical task which encourage students to be successful at mathematical tasks and mathematical achievement. This success also tends to increase students' attitudes toward mathematics in a positive manner which leads the cycle to go on and on (Koshy, Ernest & Casey, 2009).

The anticipation of the results from TIMSS 2015, and the importance of affective variables in mathematics, these factors should be

investigated. The affective variables of attitudes and attributes associated with them, such as students' confidence in mathematics, whether they like mathematics, their engagement in mathematics lessons, and whether they value mathematics should be explored for their impact on student performance in TIMSS 2015. It is necessary to investigate the characteristics of the Saudi students compared to students in the comparison countries, which may help explain the declining scores and identify factors that may improve the performance of Saudi students in upcoming TIMSS rounds.

Students' attitudes towards mathematics emerge through enjoyment of mathematics, motivation to participate in mathematics activities, and the belief that mathematics is useful, important, and worthy of trust (He, 2007). Along the same lines, the National Research Council (National Research Council, & Mathematics Learning Study Committee, 2001) noted that students' productivity desire, represented by their attitude or tendency towards mathematics, is one of the most important factors for their success in learning mathematics. High achieving students in mathematics have a set of positive attitudes and beliefs that support their learning. When students see that mathematics has value, is useful, and is worthy of attention, and also believe in the need for hard work, this contributes to learning mathematics.

According to Mullis, Martin, Foy, & Hooper (2016), TIMSS data provided an important way to understand students' attitudes towards mathematics, given the relationship between students' attitudes and their level of achievement. There is consensus that cognitive goals "intellectual side of learning" should be considered together with affective goals" objectives relating to interest, attitude, and values", suggesting the belief that these goals do not operate independently.

The Partnership for 21st Century Skills has indicated that learner attitudes and motivation are among the components that must be addressed for future learning (Beers, 2011). When planning teaching and learning processes, attention should be given to what can be done to encourage positive attitudes towards mathematical content and the appreciation of the value of mathematics, with teaching designs that draw students' attention, inspire them to learn mathematics, and convince them of its importance.

Several studies have examined students' attitudes and how they relate to performance in the TIMSS. Yavuz, Demirtasli, Yalcin, & Dibek (2017), investigated the relationship between student attributes (including feeling confident in mathematics, valuing mathematics, and attitudes towards the subject) and TIMSS scores in Turkey. The result showed a positive relationship between students' confidence in and attitudes towards mathematics and their level of achievement in TIMSS 2011. However, this was not the case in TIMSS 2007, nor was there a

relationship between students who value mathematics and the level of their performance in TIMSS in 2007 and 2011. Similarly, Bilican, Demirtasli, & Kilmen (2011) compared the attitudes towards mathematics of Turkish students who participated in TIMSS 1999 with those who participated in 2007. The results showed that the students had positive attitudes towards mathematics and that there was a relationship between their attitudes and their level of performance on the tests. In addition, the attitudes towards mathematics in 2007 were more positive than in the 1999 round. A similar study in Turkey conducted by Dogan and Baris (2010) aimed to relate some student characteristics, such as attitudes towards mathematics and whether they value mathematics, with self-efficacy among students who participated in TIMSS 1999 and 2007. The results showed a relationship between self-efficacy and student attitudes/degree of value for the two rounds.

Both He (2007) and Haciomeroglu (2017) noted that research has failed to discover consistent results on the relationship between achievement and attitudes towards mathematics. For example, Bouchey and Harter (2005), Foire (1999), Samuelsson and Granström (2007) found that negative attitudes towards mathematics often led to poor engagement and decreased student performance. On the other hand, Phan (2008), in a comparison of the performance of students in the USA, Canada, Egypt and South Africa, found that Egyptian students had high positive attitudes despite their low level of performance. In an attempt to explain this relationship, Phan indicated that such features may be related to the culture of a society, especially in developing countries, if attitudes tend to rise, regardless of achievement level.

Miller (2016) investigated some variables related to student attitudes, such as Confidence in mathematics, valuing of mathematics, and other variables related to the schools in the USA, Ghana and China. The results showed a statistically significant relationship between these variables and student achievement. Nour Al-Din and Nasser (2017) also investigated student attitude towards mathematics and its relationship to achievement to suggest a guidance programme to change negative trends in Algeria. The findings showed that a low score on the scale of attitudes was accompanied by a decrease in students' performance on a mathematics achievement test.

Some studies have been conducted on a national level that dealt with student attributes related to attitudes. Jaafari (2010) identified personal and family characteristics of students "Student attitudes - personality - performance - study habits – parental involvements" that explained the variation in performance of countries with high achievement (China and Singapore) and students of countries with low achievement (Saudi Arabia) in TIMSS 2007. The findings showed that Saudi students have positive attitudes toward mathematics in addition

to trusting in their abilities. Al-Shamrani (2010) also conducted an analytical study of the results of Saudi students' engagement in TIMSS 2007 that showed a positive relationship between students' attitudes towards mathematics and their achievement.

It seems that the real challenge for any programme seeking to develop mathematics education is to promote and maintain positive attitudes that may play a role in the development of learning outcomes. Moreover, if students want to develop a high level of performance in mathematics, they must realise that mathematics is learnable, useful, and worthy of perseverance (National Research Council, & Mathematics Learning Study Committee, 2001).

Based on the previous discussion, and due to the scarcity of research investigating how each attitude scale affects student mathematics achievement, a research problem has emerged. The goal of this paper is to determine the attitudes of eighth-grade students in the comparative countries that affect their mathematics achievement in TIMSS 2015.

Research Questions

This study is guided by the following questions:

Q1: Which one of these attitudes scales (*students´ views on engaging in mathematics lessons, students´ like learning mathematics, students´ confident in mathematics, students´ value mathematics*) are more able to explain the variation in the level of mathematical achievement among eighth graders according to the results of TIMSS 2015 in: Korea, Saudi Arabia, Singapore, America?

Q2: Does the mathematical achievement of eighth graders differ according to the difference between levels in each of the attitude scales (*students´ views on engaging in mathematics lessons, students´ like learning mathematics, students´ confident in mathematics, students´ value mathematics*) according to the results of TIMSS 2015 in each of: Korea, Saudi Arabia, Singapore, America?

Methods

Study Population and Sample

The study population consisted of all eighth-grade students (both genders) in the selected countries who participated in the mathematics test in TIMSS 2015. The sample included all members of the study population. The sample for the present study consisted of all data reported for the 8th-grade Korean, Saudi Arabian, Singaporean, and U.S. students who took the TIMSS 2015 assessment. The sample comprised 5302 students for Korea; 3574 students for Saudi Arabia; 6077 students for Singapore; and 9978 students for the U.S.

Measures

The eighth-grade mathematics achievement test scores and attitude mathematics scales were used as measures**.** The achievement scale was based on items involving content (Numbers, Algebra, Geometry, and Data and Probability) and cognitive (Knowing, Applying, and Reasoning) domains in mathematics. Student questionnaires

were intended to provide information about the attitudes of the students toward learning. Each student participating in the TIMSS answered the questions within 15-30 minutes. Questionnaire items explored the students' willingness to learn, motivation, self-concept, and related characteristics, including basic demographic information.

As aggregations of individual student scores can lead to seriously biased estimates of population characteristics (Wingersky, Kaplan, & Beaton, 1983), plausible values were used for student mathematics achievement scores (Achievement code: BSMMAT01 (Plausible Value 1), BSMMAT02 (Plausible Value 2), BSMMAT03 (Plausible Value 3), BSMMAT04 (Plausible Value 4), BSMMAT05(Plausible Value 5)).

Students' views on engaging in mathematics lessons scale. The scale was based on ten items (BSBM18A to BSBM18J). All items were rated on a 4-point Likert-type scale, ranging from '1' (Disagree a lot) to '4' (Agree a lot). The Cronbach's alpha reliability coefficients for the scale were .92, .91, .92, and .94 for South Korea, Saudi Arabia, Singapore, and the USA, respectively.

Students like learning mathematics scale. This scale was developed to measure students' interest in mathematics and how much they like learning mathematics. The scale was based on nine items (BSBM17A to BSBM17I). All items were rated on a 4-point Likert-type scale, ranging from '1' (Disagree a lot) to '4' (Agree a lot). The Cronbach's alpha reliability coefficients for the scale were 0.91, 0.75, 0.91 and .90 for South Korea, Saudi Arabia, Singapore, and the USA, respectively.

Students confident in mathematics scale. This assesses students' self-confidence or self-concept in their ability to learn mathematics. The scale was based on eight items (BSBM19A to BSBM19I). All items were rated on a 4-point Likert-type scale, ranging from '1' (Disagree a lot) to '4' (Agree a lot). The Cronbach's alpha reliability coefficients for the scale were .91, .75, .91, and .90 for South Korea, Saudi Arabia, Singapore, and the USA, respectively.

Students value mathematics scale. This scale addresses students' attitudes about the importance and usefulness of the subject, called attainment value and utility value (Wigfield & Eccles, 2000). The scale was based on nine items (BSBM20A to BSBM20I). All items were rated on a 4-point Likert-type scale, ranging from '1' (Disagree a lot) to '4' (Agree a lot). The Cronbach's alpha reliability coefficients for the scale were 0.88, 0.89, 0.87, and 0.89 for South Korea, Saudi Arabia, Singapore, and the USA, respectively.

Data Analysis

A multiple linear regression analysis was conducted to examine contributions of each attitude scale to students' mathematics achievement in each country. Furthermore, follow-up F-tests and post hoc procedures were performed to investigate significant differences in mathematics achievement among groups in each country attributed to each of the attitude scales. Those analyses were conducted by utilising the

International Association for the Evaluation of Educational Achievement (IEA) International Database Analyzer (IDB Analyzer) which was developed to analyse data from IEA surveys to ensure the appropriate use of the complex plausible value technology (IEA IDB Analyzer, 2015). For the mathematics achievement test scores, the IEA IDB Analyzer for TIMSS, a plug-in for SPSS, was used to combine the five plausible values as well as to produce their mean values and corrected standard errors. SPSS was run to compute one of the alternative F tests, Welch's test and the Games-Howell test for post hoc test procedures. Welch's test performs the best in three-group heterogeneity cases when data are normal and of unequal sample sizes (Moder, 2010).

Results

First Question:

Which one of these attitudes scales (*students' views on engaging in mathematics lessons, students' like learning mathematics, students' confident in mathematics, students' value mathematics*) are more able to explain the variation in the level of mathematical achievement among eighth graders according to the results of TIMSS 2015 in: Korea, Saudi Arabia, Singapore, America?

For the sample distribution attributed to students' mathematics achievement, the Grade 8 Singaporean student mathematics average score was the highest among all participating countries (M = 621.07, SD = 82.05) and the average score of the Korean students was the 2nd highest (M = 605.82, SD = 85.26) in TIMSS 2015. The U.S. student mathematics average score was slightly higher than average (M = 519.74, SD = 82.62) of the overall achievement distribution in 2015 and the mean of the Saudi Arabian student mathematics scores was below the average (M = 396.43, SD = 85.98) among participating countries.

Multiple regression analysis showed that for students from each of the four countries, the attitudes towards mathematics contributed significantly to the model (see Table 1). Altogether, the attitudes towards mathematics explained 32%, 14%, 17%, and 20% of the variance in Korean, Saudi Arabian, Singaporean, and U.S. students' mathematics achievement scores, respectively. The result showed significant standardised β weights for students who were engaged in their mathematics lessons in all four countries. However, it showed negative but relatively small predictive effects on mathematics achievement. Standardised β values of the like learning mathematics variable were significantly associated with mathematics achievement in Singapore (0.11), but there was a negative predictive effect in Saudi Arabia (-0.07). The β value of the like learning mathematics variable was not significant in Korea or in the USA. All β values of the confidence in mathematics variable for the four countries appear to make a significant contribution to mathematics achievement (.44 for Korea, .40 for Saudi Arabia, .36 for Singapore,

and 0.46 for the USA). β values for the value mathematics variable appeared to be significant in all countries except Saudi Arabia. The effects appear to be small for the Korean (0.2) and U.S. (.04) student samples but showed relatively small negative effects on achievement in Singapore (-.07)

Table 1

Association between mathematics attitudes dimensions and mathematics achievement in each country

	South Korea,		Saudi Arabia		Singapore		USA	
	b (s.e)	*β (s.e.)*	*b (s.e.)*	*β (s.e)*	*b (s.e.)*	*β (s.e.)*	*b (s.e.)*	*β (s.e.)*
Constant	338.55 (9.35)		199.04 (16.72)		479.87 (13.69)		370.47 (7.95)	
Engaging in Mathematics Lessons	-1.89* (0.99)	-.03* (0.02)	-0.5* (1.29)	-.01* (.03)	-0.89* (1.11)	-.02* (.02)	-2.31* (.77)	-.06* (.02)
Like Learning Mathematics	.57 (1.15)	0.01 (.02)	-3.03* (1.22)	-.07* (.03)	4.76* (1.04)	0.11* (0.02)	-1.51 (0.8)	-0.04 (.02)
Confident in Mathematics	20.07* (1.03)	0.44* (0.02)	19.30* (1.78)	0.40* (.03)	13.5* (0.82)	0.36* (.02)	16.39* (0.6)	0.46* (.01)
Value Mathematics	10.26* (1.05)	0.2* (.02)	0.88 (1.34)	0.02 (.04)	-3.12* (0.87)	-.07* (.02)	1.78* (0.69)	.04* (.02)
Adjusted R^2	.32*		.14*		.17*		.20*	

*p < 0.05

Second Question:

Does the mathematical achievement of eighth graders differ according to the difference between levels in each of the attitude scales (*students' views on engaging in mathematics lessons, students like learning mathematics, students' confident in mathematics, students' value mathematics*) according to the results of TIMSS 2015 in each of: Korea, Saudi Arabia, Singapore, America?

For each component of the attitudes toward mathematics (as shown in Table 2), students who were in higher level groups in: Engaging in Mathematics Lessons, Like Learning Math, Confident in Math, and Value Mathematics; Their average achievement scores were higher than their counterparts in the four countries. However, for each component of the attitudes toward mathematics, the percentage of students' number in each level were not equivalent. For example, only 8% of the Korean

eighth-grade students reported being very engaged during mathematics lessons compared with 52% reported being engaged in mathematics lessons and 40% reported being less engaged in mathematics lessons. And only 12% of the Saudi eighth-grade students reported being very Confident in Math compared with 50% reported being Confident in Math and 37% reported being less Confident in Math. This variations of students' number in some levels might affect the homogeneity assumption.

Table 2

The Sample Distribution of Students Views on Engaging and Attitudes toward Mathematics According to TIMSS 2015

Students' Views on Engaging in Mathematics Lessons												
Country	**Very Engaging**				**Engaging**				**Less than engaging**			
	Percent of Students		Average Achievement		Percent of Students		Average Achievement		Percent of Students		Average Achievement	
Singapore	33	(1.0)	633	(3.6)	52	(0.8)	620	(3.4)	16	(0.8)	596	(6.3)
S. Korea	8	(0.5)	642	(5.0)	52	(1.2)	614	(3.2)	40	(1.4)	589	(2.7)
USA	43	(1.2)	530	(3.5)	36	(0.7)	515	(3.3)	21	(1.0)	504	(4.0)
Saudi	50	(1.7)	376	(4.8)	35	(1.1)	366	(5.5)	15	(1.0)	349	(6.4)
Students Like Learning Mathematics												
Country	**Very Much Like Learning Mathematics**				**Like Learning Mathematics**				**Do Not Like Learning Mathematics**			
	Percent of Students		Average Achievement		Percent of Students		Average Achievement		Percent of Students		Average Achievement	
Singapore	24	(0.7)	654	(3.2)	42	(0.8)	625	(3.5)	33	(0.8)	592	(4.3)
S. Korea	8	(0.4)	668	(4.2)	34	(0.7)	634	(3.0)	58	(0.8)	581	(2.7)
USA	17	(0.6)	554	(4.0)	36	(0.6)	528	(3.4)	47	(0.9)	499	(3.0)
Saudi	21	(1.1)	396	(6.0)	37	(1.1)	370	(5.4)	42	(1.7)	354	(4.9)
Students Confident in Mathematics												
Country	**Very Confident in Mathematics**				**Confident in Mathematics**				**Not Confident in Mathematics**			
	Percent of Students		Average Achievement		Percent of Students		Average Achievement		Percent of Students		Average Achievement	
Singapore	13	(0.5)	675	(3.0)	41	(0.7)	642	(2.8)	46	(0.8)	588	(4.0)
S. Korea	8	(0.4)	687	(4.9)	38	(0.7)	643	(2.8)	55	(0.8)	569	(2.7)
USA	21	(0.7)	573	(3.5)	40	(0.6)	530	(3.0)	39	(0.9)	480	(2.9)
Saudi	12	(0.9)	433	(7.8)	50	(1.3)	373	(4.4)	37	(1.5)	342	(5.4)
Students Value Mathematics												
Country	**Strongly Value Math**				**Value Mathematics**				**Do Not Value Mathematics**			
	Percent of Students		Average Achievement		Percent of Students		Average Achievement		Percent of Students		Average Achievement	

Singapore	34	(0.8)	629	(3.5)	58	(0.7)	621	(3.4)	8	(0.4)	590	(5.8)
S. Korea	13	(0.6)	656	(4.4)	63	(0.9)	614	(2.8)	24	(0.8)	557	(3.7)
USA	44	(0.8)	531	(3.6)	45	(0.6)	516	(3.1)	11	(0.4)	488	(3.8)
Saudi	42	(1.4)	379	(5.4)	42	(1.0)	369	(4.8)	15	(0.9)	344	(7.2)

Mathematics achievement differences attributed to students' views on engaging in mathematics lessons. To examine if there is any significant difference in student mathematics achievement scores attributed to students' views on engagement in mathematics lesson scales, we used the obtained Welch's adjusted F ratio and Games-Howell post hoc tests since the assumption of homogeneity of variance has been violated. In the four countries, the one-way ANOVA of students' mathematics scores revealed a statistically significant main effect: Welch's F (2, 1170.957) = 94.411, $p < 0.001$, est. $\omega^2 = .034$ for Korea; Welch's F (2, 1593.594) = 14.309, $p < .001$, est. $\omega^2 = .007$ for Saudi Arabia; Welch's F (2, 2456.370) = 52.355, $p < 0.001$, est. $\omega^2 = .017$ for Singapore; and Welch's F (2, 5538.136) = 65.232, $p < 0.001$, $\omega^2 = .013$ for the U.S. The estimated omega squared indicated that approximately 3.4%, 0.7%, 1.7%, and 1.3% of the total variance in student mathematics scores is accounted for by differences between the levels of students' views on engagement in mathematics lesson scale for Korea, Saudi Arabia, Singapore, and the USA, respectively. The effect sizes for Korea and Singapore appear to be large (est. $\omega^2 > .014$ (Field, 2013)).

Table3

Games-Howell for the difference in the achievement of mathematics by students engaging in mathematics lessons

COUNTRY	Level	Engaging Mean Difference	Less than engaging Mean Difference
South Korea	Very engaging	29.57*	53.58*
	Engaging		24.00*
Saudi Arabia	Very engaging	8.80*	22.83*
	Engaging teaching		14.03*
Singapore	Very engaging	12.28*	35.33*
	Engaging teaching		23.04*
USA	Very engaging	13.46*	24.48*
	Engaging		11.02*

*$p < .001$

Post hoc comparisons using the Games-Howell post hoc procedure were conducted to determine which pairs of the level means differed significantly. These results are given in Table 3 and indicate that students who reported being very engaged during mathematics lessons had a significantly higher average mathematics score than students who reported being engaged and less engaged. Also, students who reported being engaged during mathematics lessons had a significantly higher average mathematics score than students who reported being less engaged.

Mathematics achievement differences attributed to how much students like learning mathematics. As for the differences in student mathematics achievement scores attributed to the Students Like Learning Mathematics Scale, the one-way ANOVA student mathematics scores revealed a statistically significant main effect for all four countries: Welch's F (2, 1201.169) = 419.924, p < 0.001 for Korea; Welch's F (2, 1899.081) = 48.598, p < .001 for Saudi Arabia; Welch's F (2, 3712.430) = 273.515, p < 0.001 for Singapore; and Welch's F (2, 4670.362) = 338.158, p < 0.001 for the USA. The estimated omega squared (ω^2 = .136 for Korea; ω^2 = .026 for Saudi Arabia; ω^2 = .082 for Singapore; and ω^2 =. 063 for the USA) indicated that approximately 13.6%, 2.6%, 8.2%, and 6.3% of the total variation in student mathematics scores is accounted for by differences between the levels of the Like Learning Mathematics Scale in Korea, Saudi Arabia, Singapore, and the USA, respectively.

Table 4

Games-Howell for the difference in achievement according to how much students like learning mathematics

COUNTRY	Level	Like learning math	Do not like learning math
South Korea	Very much like learning math	34.14*	87.09*
	Like learning math		52.95*
Saudi Arabia	Very much like learning math	22.21*	39.27*
	Like learning math		17.05*
Singapore	Very much like learning math	30.13*	61.59*
	Like learning math		31.46*
USA	Very much like learning math	26.66*	54.82*
	Like learning math		28.16*

***P < .001**

According to Games-Howell test (as shown in Table 4), students who reported being very much like learning mathematics had a significantly higher average mathematics score compared with students who reported being liking and do not like learning mathematics. Similarly, students who reported being liking learning mathematics had a significantly higher

average mathematics score compared with students who do not like learning mathematics.

Mathematics achievement differences attributed to students' confidence in mathematics. In terms of the differences in student mathematics achievement scores attributed to the Confidence in Mathematics Scale, the one-way ANOVA student mathematics scores revealed a statistically significant main effect for all four countries: Welch's F (2, 1086.426) = 899.402, $p < 0.001$ for Korea; Welch's F (2, 1224.410) = 208.914, $p < .001$ for Saudi Arabia; Welch's F (2, 2462.006) = 639.621, $p < 0.001$ for Singapore; and Welch's F (2, 5477.773) = 1134.450, $p < 0.001$ for the USA. The estimated omega squared ($\omega^2 = .253$ for Korea; $\omega^2 = .104$ for Saudi Arabia; $\omega^2 = .163$ for Singapore; and $\omega^2 = .185$ for the USA) indicated that approximately 25.3%, 10.4%, 16.3%, and 18.5% of the total variation in student mathematics scores is attributable to differences between the levels of student confidence in mathematics scale in Korea, Saudi Arabia, Singapore, and the USA, respectively. The effect sizes for the four countries appear to be large (est. $\omega^2 > .014$ (Field, 2013)).

Table 5

Games-Howell test for the difference in achievement according to students' confidence in mathematics

COUNTRY	Level	Confident in math	Not confident in math
South Korea	Very confident in math	43.14*	118.41*
	Confident in math		75.27*
Saudi Arabia	Very confident in math	58.26*	88.65*
	Confident in math		30.39*
Singapore	Very confident in math	34.78*	88.97*
	Confident in math		54.19*
USA	Very confident in math	42.07*	92.03*
	Confident in math		49.90*

***p < .001**

According to Games-Howell test (as shown in Table 5), in all counties, students who reported feeling very confident in mathematics had a significantly higher average mathematics score than students who reported feeling somewhat confident or not confident in mathematics. Similarly, students who reported feeling confident in mathematics had a significantly higher average mathematics score

than students who reported feeling not confident in mathematics.

Mathematics Achievement Differences Attributed to How Much Students Value Mathematics. For the differences in student mathematics achievement scores attributed to the Students Value Mathematics Scale, the one-way ANOVA student mathematics scores revealed a statistically significant main effect for all four countries: Welch's F (2, 1686.810) = 399.009, $p < 0.001$ for Korea; Welch's F (2, 1527.012) = 20.869, $p < .001$ for Saudi Arabia; Welch's F (2, 1347.630) = 43.620, $p < 0.001$ for Singapore; and Welch's F (2, 3123.598) = 118.161, $p < 0.001$ for the USA. The estimated omega squared ($\omega 2 = .131$ for Korea; $\omega 2 = .011$ for Saudi Arabia; $\omega 2 = .014$ for Singapore; and $\omega 2 = .023$ for the USA) indicated that approximately 13.1%, 1.1%, 1.4%, and 2.3% of the total variation in student mathematics scores is attributable to differences between the levels of the Students Value Mathematics Scale for Korea, Saudi Arabia, Singapore, and the USA, respectively. The effect sizes for Korea, Singapore, and the USA appear to be large ($\omega 2 > .014$ (Field, 2013)).

Table 6

Games-Howell test for the difference in achievement according to how much students value mathematics

COUNTRY	The level	Value math	Do not value math
South Korea	Strongly value math	42.04*	96.42*
	Value math		54.38*
Saudi Arabia	Strongly value math	10.65*	31.08*
	Value math		20.43*
Singapore	Strongly value math	9.71*	40.87*
	Value math		31.16*
USA	Strongly value math	14.66*	41.43*
	Value math		26.76*

***p < .001**

According to Games-Howell test (as shown in Table 6), in all counties, students who strongly value mathematics had a significantly higher average mathematics score than students who somewhat value or do not value mathematics. Similarly, students who value mathematics had a significantly higher average mathematics score than students who do not value mathematics.

Discussion

The result indicated that the effect of the combined attitudes on mathematical achievement was significant at the alpha level ($p \leq 0.05$) in all countries of comparison, and this may be attributed to the productive desire of the

students represented by their orientation toward mathematics one of the most important factors for their success in learning mathematics. Successful students have a set of positive attitudes and beliefs that support their learning. When students see mathematics as valuable, useful and worthwhile; This will contribute to their learning of mathematics effectively (NRC, 2001). This is consistent with the study of Lay (2017), which concluded that the positive attitudes towards mathematics of Singaporean and Malaysian students who participated in the TIMSS 2007 study played a major role in shaping mathematics achievement.

The current study also indicated that Korean eighth graders' attitudes towards mathematics have a significant effect ($f2 = 0.42$) on their mathematics achievement, in general, compared to students of the other three participating countries ($f2 < 0.15$) (Cohen, 1988), which means that the general attitude towards mathematics was effective in predicting the mathematics achievement of Korean eighth graders compared to the other three countries. In the case of South Korea, this finding could be attributed to the fact that the country works to enhance students' attitudes by supporting the spread of STEM education from elementary grades to twelfth grade (Geesa, Izci, Song, & Chen 2019). It can also be justified due to the supplemental classes in mathematics offered to Korean students outside of school time (Leung, 2017), which might help enhance their attitudes toward mathematics and increase the level of their mathematics achievement.

The result also indicated that the confidence variable in mathematics had the most effect among the other variables in all countries of comparison. This may be attributed to the strong connection between confidence and mathematical achievement. Confident students usually do well in mathematics and are able to solve complex mathematical tasks. (Koshy, Ernest & Casey, 2009)

Further, the results of the one-way ANOVA of the variable of engagement in the teaching of mathematics showed that students who were more engaged in teaching mathematics lessons had higher mathematics achievement compared to students who had a lower level of engagement in all the countries of comparison. However, the multiple regression analysis showed controversial results in all four countries, where students' engagement in teaching mathematics lessons was negatively related to their mathematics achievement. This controversial relationship could be attributed to the nature of the MR test which takes all variables in consideration as continues variables where the Anova test only takes this variable by itself. This finding is consistent with the results of the study by Lay (2017). Utilising ANOVA, Lay found that the Malaysian, Singaporean, and Thai students who had a high engagement in science classes had higher educational achievement compared to students with a lower level of engagement.

Furthermore, using multiple regression analysis, the same study reported a negative relationship between students' engagement in science lessons and their achievement in science in the TIMSS 2015 study. This may be attributed to the difference in the nature of the utilised statistical method, as the one-way ANOVA explores the effect of the *engagement in mathematics lessons* variable alone, while the multiple regression analysis deals with the effect of the variable with other components of the attitude towards mathematics achievement. The result 'regarding students' engagement' is also consistent with the study of Yildirim & Bilican (2014), which showed a negative relationship between the Turkish students' engagement in mathematics lessons and their mathematics achievement in the TIMSS 2011 study.

Moreover, the results showed that Singaporean eighth graders' views on how much they like learning mathematics is positively correlated with their mathematics achievement, and this result is consistent with the findings of Kim Park, Park, & Kim (2013), which concluded that the like learning mathematics variable has a positive effect on the mathematics achievement of Singaporean eighth graders in the TIMSS 2011 study. This also supports the results of the study by Ker (2017), which showed that attitudes are the variable with the highest impact on the mathematics achievement of Singaporean students in the TIMSS 2011 study. This may be due to the fact that students who like learning mathematics allocate more time to studying, which leads to improvement of their level of mathematics achievement.

The results also showed that Saudi students' love of learning mathematics has a negative relationship with their mathematics achievement, which is consistent with the results of Jaafari's study (2010) that concluded that there is a negative correlation between Saudi students' attitudes towards mathematics and their mathematics achievement in the TIMSS 2007 study. Similarly, the current findings are in line with that of Phan's study (2008) which compared students' performance in the United States, Canada, Egypt, and South Africa, and showed that Egyptian students have highly positive attitudes despite their low level of performance. Such findings can be explained by the connection between the attitudes and the culture of the community, such as parental involvement or it can be explained by the nature of the test where Hawthorne effect can be present, especially in developing countries, where attitudes tend to rise regardless of the level of achievement.

The value mathematics variable among eighth-grade Korean and American students also significantly correlated positively with their mathematics achievement at a significant level equal to 0.05. This finding is consistent with the findings of Kim et al. (2013), which indicated that the value mathematics variable has a positive effect on mathematics achievement of Korean fourth and eighth graders in the TIMSS 2011

study. This could be explained by the fact that students who value mathematics highly pay more attention to it, which leads to improvement of their mathematics achievement. However, it was found that Singaporean students who value mathematics have a negative correlation with their mathematics achievement, which differs from the findings of Lay (2017), which showed that a positive view of the value of mathematics held by Singaporean students participating in the TIMSS 2007 study plays a major role in shaping their mathematics achievement. A possible explanation of this could be the changes in students' perceptions of the importance of mathematics in everyday life and their future careers regardless of their level of performance in mathematics (Yavuz et al., 2017). The reason for this may also be the fact that the values and beliefs in surveys based on TIMSS data may provide an inconsistent view from one cycle to the next regarding their relationship to achievement, since it has an emotional nature and is subject to change (Dogan & Baris, 2010).

Another interesting finding of the current study was related to students' confidence in mathematics. It was found that eighth-grade students' confidence in mathematics is positively correlated with their mathematics achievement in all countries of comparison. The result of the current study may be attributed to the students' confidence in their mathematical abilities that breaks their fear barrier and reduces their anxiety toward mathematics which leads to improve the level of mathematics achievement. This result is consistent with the findings of Kim et al. (2013), which stated that confidence in mathematics has a positive effect on the mathematics achievement of fourth- and eighth-grade students in Finland, Singapore, and Korea in the TIMSS 2011 study. It is also consistent with Miller's study (2016), which showed a relationship between American students' confidence in mathematics and their mathematics achievement in the TIMSS 2011 study. The findings of the current study confirm the importance of students' confidence in their mathematics abilities, which could improve their mathematics achievement.

Conclusion

Although the literature emphasises the importance of students 'attitudes towards mathematics, and their impact on their mathematics achievement, scientific research has not been conclusive on this topic. He (2007) and Haciomeroglu (2017) noted that the results of the literature are not always consistent when it comes to the relationship between the attitudes towards mathematics and mathematics achievement. Also, Bouchey and Harter (2005), Foire (1999), and Samuelsson and Granström (2007) found that negative attitudes towards mathematics often led to less participation and decreased student performance in mathematics. It is suggested that findings from these comparative studies need to be interpreted from cross-cultural perspectives (Ker, 2017). The present study further explains relations between each scale of

attitudes toward mathematics and mathematics achievement in South Korea, Saudi Arabia, Singapore, and the USA in TIMSS 2015. As this study examined only a set of student level variables (student attitude towards mathematics).

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